

Gravitomagnetic Moments and Dynamics of Dirac (spin $\frac{1}{2}$) fermions in flat space-time Maxwellian Gravity

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Abstract

The gravitational effects in the relativistic quantum mechanics are investigated in a relativistically derived version of Heaviside's speculative gravity (in flat space-time) named here as "Maxwellian Gravity". The standard Dirac's approach to the intrinsic spin in the fields of Maxwellian Gravity yields the gravitomagnetic moment of a Dirac (spin $\frac{1}{2}$) particle exactly equal to its intrinsic spin. Violation of The Equivalence Principle (both at classical and Quantum-mechanical level) in the relativistic domain has also been reported in this work.

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1 Introduction

The interaction of the spins of fundamental fields with gravitation is a problem of potential interest, not yet fully understood. The correct value of the gravitomagnetic moment associated to spin [1], for example, is a question still asking for a consistent answer [2]. Peres [3] has shown that the iterated Dirac equation, in the presence of a gravitational field, does not contain any spin-curvature coupling, in contrast with the equation of motion of a classical spinning particle and thereby obtained the gravitational gyro-magnetic ratio κ_s (i.e., the gravitational analogue of the g -factor in spin magnetic moment) of Dirac particles as zero. The authors of [4], obtained the the gravitational gyro-magnetic factor as 1 instead of 2 as found in the electromagnetic case. Many authors [2, 5, 6, 7, 8, 9, 10] have found the value $\kappa_s = g = 2$ from other considerations. Aldrovandi et al. [5] and Wald [10] define the gravitomagnetic moment associated to the macroscopic angular momentum \vec{J} of a system or a body as $\frac{1}{2}\vec{J}$, while Mashhoon [11] defines it as $2\vec{J}$. So it seems, there is no clear picture on the concept of gravitomagnetic moment although the problem of gravitational couplings of intrinsic spins of elementary particles is under constant analysis and have been investigated for a long time both in theory and experiment (in addition to the above references, see also [12, 13, 14, 15, 16, 17, 18]). In [16] Obukhov (1998) remarked that the definition and properties of a gravitational moment in a purely Riemannian space-time of Einstein's general relativity (GR) remain unclear. Similarly there also exists confusion, in the literature, regarding the correct value of the gravitomagnetic permeability (i.e., the gravitational analogue of magnetic permeability) that would be the coefficient of a gravitomagnetic force, which is velocity dependent. This confusion stems from the following considerations.

The striking formal analogy between the Coulomb's electrostatic force between two charges i.e. $\frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ and the Newtonian gravitostatic force between two masses i.e. $\frac{Gm_1 m_2}{r^2}$, suggests that the analogous quantity for electrical permittivity ϵ_0 in gravitation is ϵ_{0g} :

$$\epsilon_{0g} = \frac{1}{4\pi G} \quad (1)$$

Since any relativistic field theory of gravity would require the existence of finite velocity of propagation of gravitational influences i.e. gravitational waves moving at the speed of light, one can deduce by analogy with electromagnetism (where the speed of electromagnetic waves $c = (\epsilon_0 \mu_0)^{-1/2}$), that the corresponding gravitational permittivity and permeability are related by $c = (\epsilon_{0g} \mu_{0g})^{-1/2}$. This implies that the gravitational or gravitomagnetic permeability must be given by [19, 20] :

$$\mu_{0g} = \frac{4\pi G}{c^2} \quad (2)$$

and this would be the coefficient of a gravitomagnetic force, which is velocity dependent. General relativity (GR) predicts [1, 21, 22] the gravitomagnetic field $\vec{B}_{g,GR}$ of a spherical spinning body (such as the Earth) under slow rotation (i.e. spinning) and weak field approximation at

$$\vec{B}_{g,GR} = 2G[\vec{J}r^2 - 3(\vec{J} \cdot \vec{r})\vec{r}]/(c^2r^5) \quad (3)$$

where \vec{J} is the macroscopic spin-angular momentum of the spinning Earth and other symbols have their usual meanings. Now, in analogy with electromagnetism, introducing the definition of gravitomagnetic moment $\vec{\mu}_g$ of a localized mass current distribution having angular momentum \vec{J} [5, 10]:

$$\vec{\mu}_g = \frac{\vec{J}}{2}, \quad (4)$$

Eq.(3) can be rewritten as

$$\vec{B}_{g,GR} = 4G[\vec{\mu}_gr^2 - 3(\vec{\mu}_g \cdot \vec{r})\vec{r}]/(c^2r^5) \quad (5)$$

Eq.(5) is formally analogous to the magnetic induction field \vec{B} produced by a localized current distribution :

$$\vec{B} = -\mu_0[\vec{\mu}r^2 - 3(\vec{\mu} \cdot \vec{r})\vec{r}]/(4\pi r^5) \quad (6)$$

where μ_0 is the magnetic permeability of empty space and $\vec{\mu}$ is the magnetic moment of the system in question. Eqs.(5) and (6), however differ in respect of their signs. This is due to the fact that in electromagnetism like charges repel and the unlike ones attract under static condition, but under dynamic condition the cases are reversed, viz., like currents (i.e. parallel currents) attract and the unlike (i.e. anti-parallel) currents repel. Two magnetic poles of the same type interact repulsively under static condition and in dynamic condition they interact in the reversed order. In case of gravitation we encounter the opposite situation, viz, like masses interact attractively under static condition and by the nature of analogy between gravitational and electrical phenomena, we expect a reversed situation in the dynamic case, viz., like (i.e. parallel) mass currents should repel (as a form of anti-gravity [23, 24]) and the unlike (i.e. anti-parallel) mass currents should attract each other. Analogously the gravitational North pole-North pole should attract [10] and the gravitational North pole-South pole should repel each other. This logical inference on the nature of gravitational interaction follows naturally from general relativity. The deep analogy between the ‘gravitomagnetic moment’ of a spinning test body in GR and the magnetic moment in electromagnetism was studied in [10].

From the formal analogy between the Eqs.(5) and (6), one can now infer the gravitational analogue of magnetic permeability μ_0 in general relativity as

$$\mu_{0g,GR} = \frac{16\pi G}{c^2} = 4\mu_{0g} \quad (7)$$

which is four times the value (Eq.(2)) expected from special relativistic consideration. R. L. Forward [23, 24] used this value (Eq.(7)) of gravitomagnetic permeability in his discussion on the velocity dependent forces in general relativity. In the post-Newtonian approximation to GR , one obtains the post-Newtonian laws of gravity [25, 26] that correspond quite closely to the Maxwellian laws of electromagnetism, from which one can also infer the relation in Eq.(7). In this setting, the speed of gravitational waves $c_{g,GR}$ in empty space in the weak field slow motion approximation limit of GR is expected (in analogy with the electromagnetic case) at

$$c_{g,GR} = (\epsilon_{0g} \cdot \mu_{0g,GR})^{-1/2} = c/2 \quad (8)$$

i.e. half the speed of light in vacuum. This expected (or unexpected) inference of the speed of gravitational waves in GR, as in Eq.(8), can be illustrated considering the following approximations to the Maxwell-type field equations of GR in the parametrized-post-Newtonian (PPN) formalism [27] (here we use somewhat different notations for the \vec{g} and \vec{H} fields of ref.[27]) :

$$\vec{\nabla} \cdot \vec{E}_g \cong -4\pi G \rho_0 \quad (9)$$

$$\vec{\nabla} \times \vec{E}_g = -\frac{1}{c} \cdot \frac{\partial \vec{B}_g}{\partial t} \quad (10)$$

$$\vec{\nabla} \cdot \vec{B}_g = 0 \quad (11)$$

$$\vec{\nabla} \times \vec{B}_g = \left(\frac{7}{2}\Delta_1 + \frac{1}{2}\Delta_2 \right) \left(-\frac{4\pi G}{c} \rho_0 \vec{v} + \frac{1}{c} \cdot \frac{\partial \vec{E}_g}{\partial t} \right) \quad (12)$$

where Δ_1 and Δ_2 are PPN parameters, ρ_0 is the density of rest masses in the local frame of the matter, \vec{v} is the ordinary (co-ordinate) velocity of the rest mass relative to the PPN co-ordinate frame. In general relativity $\left(\frac{7}{2}\Delta_1 + \frac{1}{2}\Delta_2 \right) \cong 4$ and so Eq.(12) can be rewritten as

$$\vec{\nabla} \times \vec{B}_g \cong -\frac{16\pi G}{c} \rho_0 \vec{v} + \frac{4}{c} \cdot \frac{\partial \vec{E}_g}{\partial t} \quad (13)$$

In empty space (where $\rho_0 = 0$), these field equations reduce to the following equations:

$$\vec{\nabla} \cdot \vec{E}_g = 0 \quad (14)$$

$$\vec{\nabla} \times \vec{E}_g = -\frac{1}{c} \cdot \frac{\partial \vec{B}_g}{\partial t} \quad (15)$$

$$\vec{\nabla} \cdot \vec{B}_g = 0 \quad (16)$$

$$\vec{\nabla} \times \vec{B}_g = \frac{4}{c} \cdot \frac{\partial \vec{E}_g}{\partial t} \quad (17)$$

Now taking the curl of (15) and utilizing Eqs.(14) and (17) we get the wave equation for the field \vec{E}_g in empty space as

$$\vec{\nabla}^2 \cdot \vec{E}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{E}_g}{\partial t^2} = 0 \quad (18)$$

where $c_g = c/2$. Similarly the wave equation for the field \vec{B}_g can be obtained by taking the curl of Eq.(17) and utilizing Eqs.(15) and (16) :

$$\vec{\nabla}^2 \cdot \vec{B}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{B}_g}{\partial t^2} = 0 \quad (19)$$

where again we get $c_g = c/2$. This is against the special relativistic (as well as the gauge field theoretic) expectation that the speed of gravitational waves (if they exist) should be equal to the speed of light in any Lorentz-covariant field theory of gravity. It is to be noted that the factor of 4 is responsible for this result and we shall come to the question of the origin of this factor of 4 later on.

In this paper an attempt is made to get some better understanding of these problems among others within the framework of a relativistically derived version of Heaviside's [28] speculative gravity named here as "Maxwellian Gravity". This theory is a vector model theory of gravity in Minkowski space-time having striking similarity as well as characteristic dissimilarity (due to the observed attractive nature of gravity) with Maxwell's electromagnetic theory. We named it as "Maxwellian gravity" because as far as we are aware, Maxwell first attempted to develop a vector theory of gravity, but he left it because some problem arose with which he became dissatisfied. We will discuss and attempt to resolve the problem faced by Maxwell later in this paper. Heaviside, pursued further Maxwell's left work and developed the full set of Lorentz-Maxwell-type field equations for gravity by virtue of his power of speculative thought. Unfortunately he did not work further on it for reasons unknown to us and his theory has not been studied as thoroughly as it deserves. Regrettably, the standard texts on gravitation do not contain any reference to Heaviside's work, but we came to know about his work from McDonald [28] who described Heaviside's gravity as a low velocity and weak field approximation to General Relativity. However, in this work, we shall see how the gravitational equations speculated by Heaviside can now be derived with (and also without) the aid of special relativity and see what insights come out of these equations regarding our understanding of the physical world. To attain these objectives among others, it is thought necessary to begin with a resume of the development of the field equations of Maxwellian Gravity and some of their important and immediate consequences in the following two consecutive sections before coming to the topic of this paper which is described in the subsequent sections.

2 Maxwellian Gravity

Maxwellian Gravity is a Faraday-Maxwell-type field theory of gravity in Minkowski space-time. In its formulation, the relativistic nature of gravity and the source of gravity are explored in the following line of thought.

It is well known that Newton's action-at-a distance theory of gravitation is incompatible with special relativity (SR) because it violates the principle of causality by suggesting gravitational influences propagating through space at an infinite speed. In Newton's gravitation theory, the meaning of "mass" i.e the gravitational mass [29, 30, 31], became ambiguous with the establishment of SR because SR suggests two distinct mass concepts, viz., one Lorentz-invariant rest mass [32] and other velocity dependent (i.e. frame dependent) inertial mass (better say relativistic mass) which is not Lorentz-invariant. In this setting, one key question arises, viz., what form of mass (or energy) represents the gravitational mass ? Thus in order to construct a field theory of gravity compatible with SR and the correspondence principle by which the relativistic gravity is reducible to Newtonian gravity, a decision on what "mass" (or energy) of SR represents the gravitational mass, has to be taken. Such a decision, Price [33] has rightly pointed out, will be crucial not only to the resolution of the ambiguity already mentioned but also to the issue of the non-linear nature of gravity.

In his formulation of GR ,Einstein has taken a decision in favor of the equivalence of inertial and gravitational masses which he expressed [34, 35] as :

“The proportionality between inertial and gravitational masses holds for all bodies without exception, with the [experimental] accuracy achieved thus far, so that we may assume its general validity until proved otherwise”.

Although Einstein's assumption of the proportionality of gravitational and inertial masses came out of the experimental evidence available at that time but later on it could be verified to different degrees of accuracy in numerous experiments [1, 29]. However, it is noted by Mashhoon [36] in 1993 that the observational evidence for the principle of equivalence of gravitational and inertial masses is not yet precise enough to reflect the wave nature of matter and radiation in their interactions with gravity [37, 38, 39, 40]. The equivalence of gravitational and inertial masses is the basis for Einstein's principle of equivalence between a gravitational field and an accelerated frame of reference [34, 35, 36]. It is to be noted that authors like Fock [41] and Møller [42] disagree about the equivalence of accelerated frames (or observers) and gravitational fields, while Synge [43] does not seem to believe that there is such a principle at all. The doubts generated by Fock and Synge have not disappeared completely and the equivalence principle has attracted considerable attention in the past decades; see the review [44]. Recently the validity of the equivalence

principle has also been questioned or suspected at the quantum level by many authors, see for example [12, 13, 17, 18, 45, 46]. Hammond [46] at one point, in some vein, noted : “the only thing the principle of equivalence proves is that it is wrong”. The violation of the equivalence has also been examined in the literature as a possible solution to the solar neutrino problem (see [47, 48] and the literature quoted therein). In view these controversies over the equivalence principle, let us look back closely at the theoretical logic behind Einstein’s assumption of the equality of gravitational and inertial masses. Einstein [31], by writing out the Newton’s equation of motion in a gravitational field, in full :

$$(\text{Inert mass})(\text{Acceleration}) = (\text{Gravitational mass})(\text{Intensity of gravitational field}) \quad (20)$$

inferred from it: “ *It is only when there is numerical equality between the inert and gravitational mass that the acceleration is independent of the nature of the body*”. This inference is often expressed in one of two ways :

(A_1) *that the motion of the particle is mass independent, or*

(A_2) *that the inert mass of the particle is equal to its gravitational mass.*

The two statements (A_1) and (A_2) are sometimes used interchangeably as the *weak equivalence principle* (WEP) in the literature [1, 2, 29, 30]. This use of terminology is rather confusing, as the two statements are logically independent [45]. They happen to coincide in the context of Galileo-Newtonian physics but may diverge in other settings. This is what may happen in the relativistic case as we shall see in this work. The expectation of such a possible divergence of the statements (A_1) and (A_2) stems from the observation that the above cited inference of Einstein is non-relativistic in the sense that it is drawn from a non-relativistic equation (20) and there exists the Lorentz-invariant mass (or energy) that might possibly be treated as the gravitational analogue of the Lorentz-invariant electric charge in developing a Lorentz-covariant theory of gravity. To explore and illustrate the possibility of identifying the rest mass as the gravitational analogue of the electric charge, to get new insights for developing a special relativistic generalization of Newtonian gravity, to regard old problems from a new angle, let us make a re-investigation of an often cited [20, 36] thought experiment [49] from a new angle as under.

Consider a system of two non-spinning massive point-like charged particles having such amount rest masses m_{01} and m_{02} , with respective electric charges q_1 and q_2 that they are at rest in an inertial frame K' , under equilibrium condition due to a mutual balance of the force of Coulombic repulsion and the Newtonian gravitostatic attraction between them. Our task is to investigate the condition of equilibrium the said particle system in different inertial frames in relative motion. To this end, suppose that the particles are positively charged and they are in empty space. Let the particle No.2 be positioned at the origin of K' -frame and \vec{r}_0 be the position vector of the No.1 with respect to the particle No.2. In

this frame the condition of equilibrium may be represented as

$$\vec{F}_C + \vec{F}_N = \frac{q_1 q_2 \vec{r}_0}{4\pi\epsilon_0 r_0^3} - \frac{G m_{01} m_{02} \vec{r}_0}{r_0^3} = 0 \quad (21)$$

where \vec{F}_C and \vec{F}_N respectively denotes the Coulomb and Newton force and the other symbols have their usual meanings. From Eq.(21) we get

$$q_1 q_2 = 4\pi\epsilon_0 G m_{01} m_{02} \quad (22)$$

Eq.(22) represents the condition of equilibrium, in terms of the charges and masses of the particles, under which an equilibrium will be effected in the K' -frame.

Now, in order to investigate the problem of equilibrium of the said particle system from the point of view of an observer in another inertial frame K in uniform relative motion with respect to the K' -frame and to simplify the investigation, let the relative velocity \vec{v} of K and K' -frames be along a common X/X' -axis with corresponding planes parallel as usual. Since the particles are at rest in K' -frame, both of them have the same uniform velocity \vec{v} relative to the K -frame. Let the position vector of the particle No.1 with respect to the particle No.2 as observed in K -frame be \vec{r} and the angle between \vec{v} and \vec{r} be θ .

For an observer in K -frame, the force of electric origin on either particle (say on particle No.1 due to the No.2 particle) is no more simply a Coulombic force, but a Lorentz force, viz.,

$$\vec{F}_L = q_1 \vec{E}_2 + q_1 \vec{v} \times \vec{B}_2 \quad (23)$$

where

$$\vec{E}_2 = \frac{q_2 (1 - v^2/c^2) \vec{r}}{4\pi\epsilon_0 r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} \quad (24)$$

$$\vec{B}_2 = \frac{\vec{v} \times \vec{E}_2}{c^2} \quad (25)$$

$$\vec{r} = \frac{\vec{r}_0 [1 - (v^2/c^2) \sin^2\theta]^{1/2}}{[1 - (v^2/c^2)]^{1/2}} \quad (26)$$

and the symbols have their usual meanings.

What about the force of gravitational interaction as observed in the K -frame? It can not simply be a Newtonian force but something else, otherwise the particle system will not remain in equilibrium in the K -frame. Such a situation will amount to a violation of the relativity principle of special relativity. Therefore a new force law of gravity has to be invoked so that the equilibrium is maintained in accordance with the relativity principle.

Let this new force be represented by \vec{F}_{gL} such that the equilibrium condition in K -frame be satisfied as :

$$\vec{F}_{gL} + \vec{F}_L = 0 \quad (27)$$

Taking into account the Eqs.(23)-(26), \vec{F}_{gL} in Eq.(27) can be expressed as :

$$\vec{F}_{gL} = -\vec{F}_L = -\frac{q_1 q_2 (1 - v^2/c^2)^2 \vec{r}}{4\pi\epsilon_0 r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} - \frac{q_1 q_2 (\vec{v} \cdot \vec{r})(1 - v^2/c^2) \vec{v}}{4\pi\epsilon_0 r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} \quad (28)$$

Now with the help of Eq.(22), we can eliminate $q_1 q_2$ from Eq.(28) and get the expression for \vec{F}_{gL} as :

$$\vec{F}_{gL} = -\frac{G m_{01} m_{02} (1 - v^2/c^2)^2 \vec{r}}{r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} - \frac{G m_{01} m_{02} (\vec{v} \cdot \vec{r})(1 - v^2/c^2) \vec{v}}{c^2 r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} \quad (29)$$

Eq.(29) may be rearranged into the following form :

$$\vec{F}_{gL} = m_{01} \vec{E}_{g2} + m_{01} \vec{v} \times \vec{B}_{g2} \quad (30)$$

where

$$\vec{E}_{g2} = -\frac{G m_{02} (1 - v^2/c^2) \vec{r}}{r^3 [1 - (v^2/c^2) \sin^2\theta]^{3/2}} \quad (31)$$

$$\vec{B}_{g2} = \frac{\vec{v} \times \vec{E}_{g2}}{c^2} \quad (32)$$

Eqs.(30)-(32) are in complete formal analogy with Eqs.(23)-(25) of classical electromagnetism in its relativistic version. Thus, from the requirement of the frame-independence of the equilibrium conditions, we not only obtained a gravitational analogue of the Lorentz-force law (or the gravitational Lorentz force law in Eq.(30)) but also unexpectedly found the Lorentz-invariant rest mass as the gravitational analogue of the electric charge by analogy. From this analysis, the gravitational charge (or mass) invariance may be interpreted as a consequence of the Lorentz-invariance of the physical laws. These findings are in conformity with Poincaré's [50] remark that *if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force*. Having recognized these findings we obtained four Faraday-Maxwell-type linear equations of gravity describing what we call "Maxwellian Gravity" following the known procedures of the electromagnetic theory; see for example, an excellent text by Rosser[51]. The resulting equations have a surprisingly rich and detailed correspondence with Faraday-Maxwell's field equations of the

electro-magnetic theory. The field equations can be written in the following Faraday-Maxwellian-form:

$$\vec{\nabla} \cdot \vec{E}_g = -4\pi G \rho_0 = -\rho_0/\epsilon_{0g} \quad \text{by defining} \quad \epsilon_{0g} = 1/4\pi G \quad (33)$$

$$\vec{\nabla} \times \vec{B}_g = -\mu_{0g} \vec{j}_0 + (1/c^2)(\partial \vec{E}_g / \partial t), \quad \text{by defining} \quad \mu_{0g} = 4\pi G/c^2 \quad (34)$$

$$\vec{\nabla} \cdot \vec{B}_g = 0 \quad (35)$$

$$\vec{\nabla} \times \vec{E}_g = -\partial \vec{B}_g / \partial t \quad (36)$$

Where ρ_0 = rest mass (or proper mass) density; \vec{j}_0 = rest mass current density; G is Newton's universal gravitational constant; c is the speed of light in empty space; the gravito-electric and gravito-magnetic fields \vec{E}_g and \vec{B}_g respectively are defined by the gravitational Lorentz force on a test particle of rest mass m_0 moving with uniform velocity \vec{u} as

$$\frac{d}{dt}[m_0 \vec{u} / (1 - u^2/c^2)^{1/2}] = m_0 [\vec{E}_g + \vec{u} \times \vec{B}_g] \quad (37)$$

where the symbols have their respective meanings in correspondence with the Lorentz force law in its relativistic form. Interestingly the field equations (33)-(36) happen to coincide structurally with those speculated by Heaviside [28]. In covariant formulation, introducing the space-time four vector $x_\mu = (x, y, z, ict)$, proper mass current density four vector $j_\mu = (j_{0x}, j_{0y}, j_{0z}, ic\rho_0)$ and the second-rank antisymmetric gravitational field strength tensor

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_{gz} & -B_{gy} & -iE_{gx}/c \\ -B_{gz} & 0 & B_{gx} & -iE_{gy}/c \\ B_{gy} & -B_{gx} & 0 & -E_{gz}/c \\ iE_{gx}/c & iE_{gy}/c & iE_{gz}/c & 0 \end{pmatrix} \quad (38)$$

The field equations (33-36) can now be represented by the following two equations:

$$\sum_{\nu} \partial F_{\mu\nu} / \partial x_{\nu} = -\mu_{0g} \vec{j}_{\mu}, \quad \text{where} \quad \mu_{0g} = 4\pi G/c^2 \quad (39)$$

$$\partial F_{\mu\nu} / \partial x_{\lambda} + \partial F_{\nu\lambda} / \partial x_{\mu} + \partial F_{\lambda\mu} / \partial x_{\nu} = 0 \quad (40)$$

while the gravitational Lorentz force law (37) assumes the form :

$$c^2(d^2 x_{\mu} / ds^2) = F_{\mu\nu}(dx^{\nu} / ds) \quad (41)$$

The absence of the rest mass of the test particle in its co-variant equation of motion (41) in the external gravitational field $F_{\mu\nu}$ describes clearly the universality of free fall or the uniqueness of free fall (UFF) known since the time of Galilei. Here in the relativistic

domain we saw the UFF also holds true for non-spinning particle-field system but in addition to this we also notice here a divergence of the statements (A_1) and (A_2) stated earlier after Einstein's inference from Eq.(20).

The present analysis seems to establish an equivalence of gravitational mass and rest mass (or any form of Lorentz-invariant mass-energy) and predict the existence of a magnetic-type component in gravity. Although the inertial mass of a body depends on its total energy content as revealed by special relativity, this mass seems not to represent the gravitational charge (or mass) as per the revelations of this analysis which differs from that made by the original designers of the thought experiment[47], who axiomatically used the inertial mass as the gravitational charge (or mass). However, the present derivation of the gravitomagnetic field agrees with the works of Sciama [52] and Bedford and Krumm[53] who axiomatically used the rest mass as the gravitational mass. It is to be noted that the main purpose of this analysis of the thought experiment is to illustrate how the rest mass of a particle manifests itself as the gravitational analogue of the electric charge in a specific situation. Since the rest mass is seen here as playing the role of the gravitational charge (or mass) in a specific situation, we have reasons to expect it to play the same role in other situations also even when the electric force is not balanced by a gravitational force. By assuming the equality of gravitational and rest masses, the validity of Newton's law of gravitation and the principle of Lorentz-invariance of all physical laws, the gravitational analogues of Maxwell-Lorentz equations (33)-(37) ,can also be obtained following the methods of Rosser [51] or Frisch and Wilets [54] as applied to electromagnetic theory. So one need not bother about the requirement of electromagnetic considerations or the necessity of a balance between the forces of electromagnetic and gravitational origins, in invoking the magnetic-type component in gravity. For another alternative and interesting approach to Maxwellian Gravity, Bergström's [56] approach to the origin of magnetic field is worth noting. In this approach one can describe the gravitomagnetic force as a Coriolis force resulting from Thomas rotation caused by the gravitational force. This point was transparently clear to Borgström.

Further it is to be noted that even without the aid of special relativity ,one can infer the gravitational analogues of Maxwell-Lorentz equations by combining three ingredients,viz.,
(i) the laws of gravitostatics;
(ii) the Galileo-Newton principle of relativity (masses at rest and masses with a common velocity viewed by a co-moving observer are physically indistinguishable);
(iii) the postulate on the existence of gravitational waves that travel in vacuum at a speed c_g called the speed of gravitational waves and following the approach of Schwinger et al. [55] to electromagnetic theory. The field equations that emerge from this approach coincide with the Eqs.(33)-(36) when $c_g = c$.

Judged from all these variant approaches that lead to Maxwellian Gravity, we have reasons to suspect the existing belief [1] that gravitomagnetism is a manifestation of space-time curvature as described in general relativity.

3 Consequences of the Maxwellian Gravity

Maxwellian Gravity is very much analogous to Maxwell's electromagnetic theory as revealed by the form of its equations. Therefore gravitational phenomena very much analogous to those of electromagnetic theory are not surprising to be revealed by this theory. However few concepts and results of unconventional nature and importance may be discussed as under.

Let us start with the concept of field energy density in gravitation theory. This concept is of some historical as well as physical importance in any field theory of gravity. It has been pointed out by McDonald that [28]:

“ J. C. Maxwell ended his great paper 1864 ‘A Dynamical Theory of Electromagnetic Field’ with remarks on Newtonian gravity as a vector field theory. He was dissatisfied with his results because the potential energy of a static configuration is always negative but he felt this should be re-expressible as an integral over field energy density which, being the square of the gravitational field, is positive”.

However, this dissatisfaction of Maxwell over a vector theory of gravity can be overcome if energy density of gravito-electric (i.e. the electric-type component of gravity) and gravitomagnetic (i.e. the magnetic-type component) field, respectively are defined with a negative sign in the following manner, viz.,

$$(i) \ u_{ge} = -\frac{1}{2}\epsilon_{0g}\vec{E}_g \cdot \vec{E}_g \quad (ii) \ u_{gm} = -\frac{1}{2\mu_{0g}}\vec{B}_g \cdot \vec{B}_g \quad (42)$$

where $\epsilon_{0g} = 1/4\pi G$ and $\mu_{0g} = 4\pi G/c^2$ and the total field energy density is given by a sum of the above two, i.e.

$$u_{field} = u_{ge} + u_{gm} \quad (43)$$

For a particle at rest, i.e., in gravitostatics, the only contribution to its gravitational field energy is that due to the gravito-electric field. This definition of gravitational field energy may most easily be obtained by analogy with electromagnetism, noting that the negative sign is a consequence of the attractive nature of gravity. In gravitostatics, it is easy to

compute the gravitational or gravito-electric self energy of a sphere of radius R and rest mass M_0 with uniform rest mass density by using Eq.(42i), which comes out as :

$$U_g = -\frac{1}{2} \epsilon_{0g} \int_0^\infty E_g^2 4\pi r^2 dr = -\frac{3GM_0^2}{5R} \quad (44)$$

The result (44) is in complete agreement with the Newtonian result. It is to be noted that Visser [57] used exactly this definition of gravitational field energy density in his classical model for the electron. Such a definition of the field energy of gravity has the advantage of describing the correct nature of gravitation on quantization because in analogy with electromagnetic theory Maxwellian Gravity will eventually lead to a gauge field of spin 1 and the spin 1 gauge fields having positive and definite field energy, on quantization, as we know lead to a repulsive force field for identical charges of such fields. It is due to this reason Gupta [58] suggested rejection of any spin 1 gauge theory of gravity with field energy being positive and definite as such fields do not account for the observed nature of gravitational interaction.

The law of energy-momentum conservation is one of the important aspect of the validity of any physical theory. By analyzing Einstein's general relativity, Denisov and Logunov [59] have shown that General Relativity does not obey this strict law of nature when matter and gravitational field are taken together. Inasmuch as the theories of other physical fields, a unified conservation law of energy-momentum exists for different forms of matter, and since there is at present no experimental evidence of its violation (moreover, the history of physics has always illustrated its tenacity and truth), there is no reason to reject this law. However, with the gravitational Lorentz force law (30 or 37) as revealed in this paper, the field momentum density defined by

$$\vec{N} = \vec{P}_g/c^2 = (\vec{H}_g \times \vec{E}_g)/c^2 \quad (45)$$

Where the gravitomagnetic field intensity \vec{H}_g is defined (in empty space) as

$$\vec{H}_g = \vec{B}_g/\mu_{0g} \quad (46)$$

and the gravitational Poynting vector \vec{P}_g defined as

$$\vec{P}_g = \vec{H}_g \times \vec{E}_g \quad (47)$$

and the field energy defined by (42), it is easy to verify that Maxwellian gravity is consistent with that sacrosanct law of nature. The gravitational Poynting vector defined in Eq.(47) is in agreement with that used by Krumm and Bedford [60].

In empty space the fields \vec{E}_g and \vec{B}_g of this theory satisfy the following two wave equations

$$\nabla^2 \cdot \vec{E}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{E}_g}{\partial t^2} = 0 \quad (48)$$

$$\nabla^2 \cdot \vec{B}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{B}_g}{\partial t^2} = 0 \quad (49)$$

where $c_g = c$. Thus the theory, in the spirit of Maxwell's electromagnetic theory, predicts beyond doubt the existence of gravitational waves traveling through empty space exactly at the speed of light as expected. Like electromagnetic waves these gravitational waves are transverse in nature and carry energy momentum. Further the fields \vec{E}_g and \vec{B}_g of the new theory are derivable from potential functions

$$\vec{B}_g = \nabla \times \vec{A}_g, \quad \vec{E}_g = -\nabla \cdot \Phi_g - \partial \vec{A}_g / \partial t \quad (50)$$

where Φ_g and \vec{A}_g represents respectively the gravitational scalar and vector potential of the new theory. These potentials satisfy the inhomogeneous wave equations :

$$\nabla^2 \cdot \Phi_g - \frac{1}{c^2} \cdot \frac{\partial^2 \Phi_g}{\partial t^2} = 4\pi G \rho_0 = \rho_0 / \epsilon_{0g} \quad (51)$$

$$\nabla^2 \cdot \vec{A}_g - \frac{1}{c^2} \cdot \frac{\partial^2 \vec{A}_g}{\partial t^2} = \frac{4\pi G}{c^2} \vec{j}_0 = \mu_{0g} \vec{j}_0 \quad (52)$$

if the gravitational Lorenz [61] gauge condition

$$\vec{\nabla} \cdot \vec{A}_g + \frac{1}{c^2} \frac{\partial \Phi_g}{\partial t} = 0 \quad (53)$$

is imposed. These will determine the generation of gravitational waves by prescribed gravitational charge and current distributions. Particular solutions (in vacuum) are

$$\Phi_g(\vec{r}, t) = -G \int \frac{\rho_0(\vec{r}', t')}{|\vec{r} - \vec{r}'|} dv' \quad (54)$$

$$\vec{A}_g(\vec{r}, t) = -\frac{G}{c^2} \int \frac{\vec{j}_0(\vec{r}', t')}{|\vec{r} - \vec{r}'|} dv' \quad (55)$$

where $t' = t - |\vec{r} - \vec{r}'|/c$ is the retarded time. These are called the retarded potentials. Thus we saw that retardation in gravity is possible in Minkowski space-time in the same procedure as we adopt in electrodynamics. This result seems to conflict with the view [62] that Newtonian gravity is entirely static, retardation is not possible until the correction

due to deviations from Minkowski space is considered.

Einstein's general theory of relativity (GR) also predicts such potentials in the weak field limit. But the vector potential and so the gravitomagnetic field of Einstein's theory differ (as we shall see below) from that of Maxwellian Gravity by a factor of 4. It is interesting to note that there is evidence for the existence of gravitational vector potential and magnetic-type component in gravity from Lunar Laser Ranging [63] and the LAGEOS experiment [64] on the detection of the gravitomagnetic effect of the spinning Earth. General Relativity predicts [1, 14, 15, 24] the gravitomagnetic induction field $\vec{B}_{g,GR}$ of the spinning Earth under slow rotation (spinning) and weak field approximation at a value given by Eq.(3). However, in Maxwellian Gravity following the standard electromagnetic procedure of estimation of magnetic field generated by localized current distributions (see for example, Jackson [65]), the gravitomagnetic induction field of the Earth (under slow spinning motion condition when rest mass \simeq inertial mass) can be estimated at

$$\vec{B}_g = \frac{G}{2c^2 r^5} [\vec{J} r^2 - 3(\vec{J} \cdot \vec{r}) \cdot \vec{r}] - \left(\frac{4\pi G}{3c^2} \right) \vec{J} \delta(\vec{r}) \quad (56)$$

where $\delta(\vec{r})$ is Dirac's δ -function. Thus we have (in view of (3))

$$\vec{B}_g = \frac{B_{g,GR}}{4} - \left(\frac{4\pi G}{3c^2} \right) \vec{J} \delta(\vec{r}), \quad (57)$$

and when $\vec{r} \neq 0$,

$$\vec{B}_g = \frac{\vec{B}_{g,GR}}{4} \quad (58)$$

So a satellite orbiting the Earth having a gravitomagnetic field presumed at $\vec{B}_g = (\vec{B}_{g,GR})/4$ would experience a Lense-Thirring (LT)[66] type nodal precession at

$$\dot{\vec{\Omega}}_{LT,MG} = \dot{\vec{\Omega}}_{LT}/4 = 25\% \text{ of the } LT \text{ precession} \quad (59)$$

as the Lense-Thirring nodal precession [66] is given by

$$\dot{\vec{\Omega}}_{LT} = \frac{2G\vec{J}}{c^2 a^3 (1 - e^2)^{3/2}} \quad (60)$$

where a and e respectively represents the semi-major axis and eccentricity of the satellite orbit. It is to be noted that LAGEOS experiment [64] measured the Lense-Thirring precession to an accuracy of 20% – 30% using laser ranging to two Earth satellites as per a report by Unnikrishnan [67]. Currently we are in search of an alternative explanation for the LAGEOS result within this theory and it will be addressed in our future works which

may include the planetary precession also.

Let us now come to the question of the origin of the factor of 4 in general relativistic gravitomagnetic field. The mathematical theory of General relativity is based on the concept of the equality of gravitational and inertial masses and the concept of Riemann space-time, which are absent in Maxwellian Gravity. So the origin of the factor of 4 might be at the foundation level. To get some insights of this expectation, let us consider the original analysis of the thought experiment [49], where space-time is kept Minkowskian and by assumption inertial mass is taken as the gravitational mass. From the original work of the cited thought experiment, the magnitude of the gravitomagnetic component of the force between the particles turns out, in the lowest order approximation, at

$$F_{[49]} \simeq \frac{2G m_{01} m_{02} v^2}{c^2 r^3} \quad (61)$$

But in Maxwellian gravity the same force under identical situation comes out as

$$F_{MG} \simeq \frac{G m_{01} m_{02} v^2}{c^2 r^3} \quad (62)$$

Thus the co-efficients of the two forces (61) and (62) differ by a factor of 2 and so the gravitomagnetic permeability in (61) is twice as implied by Maxwellian Gravity. In general relativity the gravitomagnetic permeability is four times as implied by Maxwellian gravity, as we have seen earlier. Since a factor of 2 originates from the incorporation of the equivalence of gravitational and inertial masses in the flat space-time relativistic gravity, the origin of another factor of 2 may be ascribed to the space-time curvature.

4 Gravitomagnetic moment, the spin and the relation between them

The classical connection between angular momentum \vec{L} and magnetic moment $\vec{\mu}_L$ (in S.I. units) :

$$\vec{\mu}_L = (q/2m)\vec{L}, \quad (63)$$

which holds for orbital motion even on atomic scale is well-known. Analogously, a classical connection between angular momentum \vec{L} and gravitomagnetic moment (the gravitational analog of magnetic moment) comes out in Maxwellian Gravity as

$$\vec{\mu}_{gL} = (m_0/2m)\vec{L} \quad (64)$$

which holds for orbital motion even on atomic scale as well because of the purely kinematic definition of gravitomagnetic moment of a particular volume containing proper mass currents $\vec{j}_0 = \rho_0 \vec{u}(\vec{r})$ defined (in Maxwellian Gravity) by

$$\vec{\mu}_{gL} = (1/2) \int (\vec{x} \times \vec{j}_0) d^3x = (1/2) \int \rho_0 (\vec{x} \times \vec{u}) d^3x \quad (65)$$

and the standard definition of mechanical angular momentum \vec{L} in terms of the velocity distribution of inertial mass densities ρ_m :

$$\vec{L} = \int \rho_m (\vec{x} \times \vec{u}) d^3x \quad (66)$$

It is to be noted that m in (63) and (64) represents the inertial mass of the system (or particle) in question which is relativistically distinct from its rest mass m_0 . The notion of gravitomagnetic moment was proposed in [68] and for other studies the reader may refer to [2, 3, 4, 5, 6, 7, 8, 9, 10, 67, 69].

From (63) and (64) we can form a ratio

$$(\mu_L/\mu_{gL}) = q/m_0 = a \quad \text{Lorentz - invariant quantity} \quad (67)$$

The relation (67) holds good irrespective of the magnitude of the inertial masses. It is well known that the classical connection (63) fails for the intrinsic magnetic moment of electrons and other elementary particles. For electrons, the intrinsic magnetic moment is slightly more than twice as large as implied by (63), with the spin angular momentum \vec{S} replacing \vec{L} and the rest mass m_0 replacing the inertial mass m . Thus we speak of the electron having a g -factor of 2(1.00116) and the spin magnetic moment $\vec{\mu}_s$:

$$\vec{\mu}_s = (gq/2m_0)\vec{S} \quad (\text{in S.I. units}) \quad (68)$$

The departure of the magnetic moment from its classical value has its origins in relativistic and quantum-mechanical effects which will be reiterated later in this paper in connection with the quantum-mechanical description of spin gravitomagnetic moment. An interesting feature of the relation (68) is that the ratio of the charge of interaction and the intrinsic mass m_o appears as a proportionality constant. If we look at the magnetic moment as the quantity that is the source of an elementary magnetic dipole field or as the quantity that describes, the response to an applied magnetic torque then the proportionality constant has the structure (charge of the field/intrinsic mass). This observation differs from that of Unnikrishnan [67] in that the proportionality constant has the structure (charge of the field/inertial mass). Our observation stems from Dirac's prediction of the spin magnetic moment of electrons. This observation allows us to advance the hypothesis that in the case of gravitational interaction, the spin angular momentum and "the gravitational spin"

(or the gravitomagnetic moment) are connected in a general way, with the proportionality containing the ratio (gravitational charge (or mass)/ intrinsic mass). The proportionality constant (gravitational mass/ intrinsic (or rest) mass) is equal to unity in the framework of Maxwellian Gravity. Now in analogy with (68) we define the spin gravitomagnetic moment $\vec{\mu}_{gs}$:

$$\vec{\mu}_{gs} = (\kappa_s/2)\vec{S} \quad (69)$$

with κ_s being the gravitational analog of the g -factor in (68). It is interesting to note that a ratio (μ_s/μ_{gs}) formed in analogy with (67) yield

$$(\mu_s/\mu_{gs}) = q/m_0 \quad (70)$$

under the condition that $g = \kappa_s$. Let us now make a quantum-mechanical investigation of this condition ($g = \kappa_s$) using Dirac's theory of electrons in flat space-time.

Dirac's equation for the motion of a charged massive particle (say electron) of rest mass m_o and charge q in an external electromagnetic field having electric and magnetic components may be written as

$$i\hbar(\partial\Psi/\partial t) = H\Psi = [c\alpha \cdot (\vec{P} - q\vec{A}_e) + \beta m_0 c^2 + q\Phi_e]\Psi \quad (71)$$

where Φ_e and \vec{A}_e represent the scalar and vector potentials of the external electromagnetic field, α and β are Dirac matrices in the representation of Bjorken and Drell[70] and other symbols have their usual meanings. If in addition to the electromagnetic field, there exists such other fields as predicted by Maxwellian Gravity, then Eq.(71) may be amended to the following generalized form

$$i\hbar(\partial\Psi/\partial t) = H\Psi = [c\alpha \cdot (\vec{P} - q\vec{A}_e - m_0\vec{A}_g) + \beta m_0 c^2 + q\Phi_e + m_0\Phi_g]\Psi \quad (72)$$

where Φ_g and \vec{A}_g represent the gravitational scalar and vector potentials of Maxwellian Gravity. Since there exists no charged particle without mass and no massive particle without creating and yielding to gravity, the generalized Dirac equation (GDE) (72) is expected to describe more correctly the interaction of Dirac particles among themselves and with gravity as it takes into account the gravitational contributions to the Hamiltonian which may not be neglected in a variety of situations in high energy physics.

The relativistic energy of the test particle either in (25) or (26) includes also its rest energy $m_o c^2$. Now following the standard procedure of obtaining Pauli's equation from (71) [70], we obtained the following generalized Pauli's equation (GPE) from the GDE (72), in the first non-relativistic approximation :

$$i\hbar \left(\frac{\partial\Psi}{\partial t} \right) = \hat{H}\Phi \simeq \left[\frac{(\vec{P} - q\vec{A}_e - m_0\vec{A}_g)^2}{2m_0} + q\Phi_e + m_0\Phi_g - \frac{q\hbar}{2m_0}\vec{\sigma} \cdot \vec{B} - \frac{\hbar}{2}\vec{\sigma} \cdot \vec{B}_g \right] \Phi \quad (73)$$

where $\vec{B} = \vec{\nabla} \times \vec{A}_e$, $\vec{B}_g = \vec{\nabla} \times \vec{A}_g$, Φ is the upper component of Dirac's bi-spinor $\Psi = \begin{pmatrix} \Phi \\ \chi \end{pmatrix}$ and other symbols have their usual meanings. The last two terms in the non-relativistic Hamiltonian (73) have the form of the potential energy of a dipole in external fields. Thus in the first approximation, the charged particle behaves as a particle having a spin magnetic moment

$$\vec{\mu}_s = \left(\frac{q\hbar}{2m_0} \right) \vec{\sigma} = (q/m_0) \vec{S} \quad (74)$$

and a spin gravitomagnetic moment

$$\vec{\mu}_{gs} = (1/2)\hbar\vec{\sigma} = \vec{S} \quad (75)$$

where $\vec{S} = (1/2)\hbar\vec{\sigma}$ is the spin angular momentum of the particle in question. The relation (74) is the well known result of Dirac's theory which when compared with (68) yields a g -factor of $g=2$. The relation (75) is our new result which when compared with (69) yields $\kappa_s = 2$.

5 Dynamics of the Dirac fermions in Maxwellian Gravity

The dynamics of the Dirac fermions in different gravitational fields and non-inertial reference frames was studied previously ([12, 14, 16, 17, 18] and the literature quoted therein) using different schemes. Here we will present some results that come in the description of the interaction of a spin 1/2 particle (Dirac fermion) with the fields of Maxwellian Gravity adopting the very procedures of the standard electromagnetic case. The correct description of the interaction of a spin 1/2 particle with an electromagnetic field is given by the effective Hamiltonian [71] derived from the Dirac equation (71) using the method of Foldy-Wouthuysen (FW) transformations [72]. In the gravitational case of Maxwellian gravity, the gravitational analog of the Dirac equation (71) is

$$i\hbar \left(\frac{\partial \Psi}{\partial t} \right) = [c\alpha \cdot (\vec{P} - m_0 \vec{A}_g) + \beta m_0 c^2 + m_0 \Phi_g] \Psi \quad (76)$$

and the corresponding effective Hamiltonian can analogously be obtained as

$$H_{eff} = \frac{(\vec{P} - m_0 \vec{A}_g)^2}{2m_0} - \frac{\vec{P}^4}{8m_0^3 c^2} + m_0 \Phi_g - \frac{\hbar}{2} \vec{\sigma} \cdot \vec{B}_g + \frac{\hbar^2 [\vec{\nabla}^2 \Phi_g]}{8m_0 c^2} + \left(\frac{\hbar}{4m_0 c^2 r} \right) \left(\frac{d\Phi_g}{dr} \right) \vec{\sigma} \cdot \vec{L} \quad (77)$$

where the various terms have their respective gravitational meanings corresponding to their electromagnetic counter parts. Hence, what we may call *gravitational fine structure* comes from three terms:

- $-\frac{\vec{P}^4}{8m_0^3c^2}$ *relativistic mass increase*;
- $\frac{\hbar^2[\nabla^2\Phi_g]}{8m_0c^2} = -\frac{\hbar^2[\vec{\nabla}\cdot\vec{E}_g]}{8m_0c^2}$ *Gravito-Darwin term*

which clearly admits a physical interpretation similar to that of the usual electromagnetic Darwin term, reflecting the *Zitterbewegung* fluctuation of the fermion's position, that make the fermion sensitive to the average gravitational potential in the vicinity of its average position. Gravitational Darwin term has already been discussed earlier [12, 16] in the description of the behavior of Dirac particles immersed in the fields of General Relativity where the gravitational Darwin term differs from the present term by a factor of 4 as expected in view of the analysis made in this paper. Hehl and Ni [73], who derived the inertial effects for a Dirac particle in accelerated and rotating frames using Minkowski space-time, have also found a term (what they call red shift to kinetic energy) that has been shown [12] to have the form of a Darwin term. Interestingly, Hehl and Ni's red shift to kinetic energy term - interpreted in [12] as the gravitational analogue of Darwin term - is four times larger than the Gravito-Darwin term we found here. This difference may be interpreted as a manifestation of the violation of the equivalence principle.

- $\left(\frac{\hbar}{4m_0c^2r}\right)\left(\frac{d\Phi_g}{dr}\right)\vec{\sigma}\cdot\vec{L} = \left(\frac{1}{2m_0c^2r}\right)\left(\frac{d\Phi_g}{dr}\right)\vec{S}\cdot\vec{L}$ *gravitational spin-orbit term.*

This spin-orbit term is due to the interaction of the fermion's gravitomagnetic moment (i.e. the spin) with the gravitomagnetic field it sees due to its motion and automatically includes the Thomas precession as in the electronic case. For recent discussions on gravitational spin-orbit coupling the reader may refer to [12, 16, 17] where the gravitational spin-orbit term is shown as the EEP (Einstein's equivalence principle) violating term. But the inertial spin-orbit coupling term, which first turned up as a result of Hehl and Ni's calculation [73] coincides with the prediction here under the condition: inertial acceleration $\vec{a} = -\vec{E}_g$. Hence we observe this term as not the EEP-violating term.

Corresponding to the full Zeeman effect (Dirac) in the electronic case, we here predict what we call the Gravito-Zeeman effect as arising out of two terms: the orbital part is $-\frac{1}{2}(\vec{P}\cdot\vec{A}_g + \vec{A}_g\cdot\vec{P}) = -\frac{1}{2}\vec{B}_g\cdot\vec{L}$, where $\vec{A}_g = -\frac{1}{2}(\vec{r}\times\vec{B}_g)$ for a weak uniform gravitomagnetic field \vec{B}_g and the spin part is $-\frac{\hbar}{2}\vec{\sigma}\cdot\vec{B}_g$. Combining the two parts we get

$$H_{gravito-Zeeman} = -\frac{1}{2}\vec{B}_g\cdot(\vec{L} + 2\vec{S}) \quad (78)$$

where $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$. Note the factor of 2 for the fermion's intrinsic gravito-gyro-magnetic ratio. From the GDE (72) or from the GPE (73) one can obtain the gravitationally modified (or generalized) Zeeman effect (GZE), in the same procedure as in the electromagnetic case, corresponding to the Hamiltonian :

$$H_{GZE} = -\frac{1}{2} \left(\vec{B}_g + \frac{q\vec{B}}{m_0} \right) \cdot (\vec{L} + 2\vec{S}) \quad (79)$$

In specific situations where $\vec{B}_g + q\vec{B}/m_0 = 0$, Zeeman effect can be nullified. In other cases Zeeman effect gets modified in presence of a gravitomagnetic field. Hence, Zeeman Effect may be employed for the detection of gravitomagnetic of Earth or other rapidly rotating astrophysical objects. Another interesting inference of this analysis is that unlike the spin magnetic moment, the spin gravitomagnetic is independent of the charge and mass of the Dirac particle. Therefore charged Dirac particles that can be easily handled electromagnetically in any desired way to move with relativistic speeds in the gravitational field of the earth, can also be employed to detect the gravitomagnetic field of the earth. Further Eq.(79) may be rewritten as

$$H_{GZE} = -\frac{1}{2} \left(\vec{B}_g + \vec{\omega}_c \right) \cdot (\vec{L} + 2\vec{S}) \quad (80)$$

where $\vec{\omega}_c = q\vec{B}/m_0$ is the cyclotron frequency. Hence here seems a possibility of utilizing the cyclotrons for the detection of the gravitomagnetic field of the Earth.

6 Concluding Remarks

For a massive charged Dirac ($spin\frac{1}{2}$) fermion we found (in Sec.3) that the gravitomagnetic moment (= the spin) is independent of its rest mass and electric charge. Hence we remark that all spin $\frac{1}{2}$ particles possessing whatever rest masses and electric charges must interact with gravity identically under identical conditions. In this sense the spin-gravity and spin-spin interaction is a universal phenomenon for all spin $\frac{1}{2}$ particles irrespective of their rest masses and electric charges. The spin-spin interaction Hamiltonian of two Dirac particles having spins \vec{S}_1 and \vec{S}_2 here comes out(in analogy with the electromagnetic case) as :

$$H_{spin-spin} = -\vec{B}_g \cdot \vec{S}_1 \cdot \vec{S}_2 = \left(\frac{8\pi G}{3c^2} \right) \vec{S}_1 \cdot \vec{S}_2 \delta(\vec{r}) - \frac{G}{c^2 r^5} [(\vec{S}_1 \cdot \vec{S}_2) r^2 - 3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r})] \quad (81)$$

where $\delta(\vec{r})$ is Dirac's δ -function. It must be noted that this is different from Lense-Thirring effect which involves the rotation of the frame of the reference. The δ -function contribution to the Hamiltonian in (81) is the gravitational analogue of hyperfine interaction as it

resembles the interaction of the spin of the electron with the magnetic field of the nucleus of an atom. This effect is very negligible in atomic cases because the coefficient of this contribution contains the ratio of G and c^2 . When $r \neq 0$ we have

$$H_{spin-spin} = -\vec{B}_g \cdot \vec{S}_2 = -\frac{G}{c^2 r^5} [(\vec{S}_1 \cdot \vec{S}_2) r^2 - 3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r})] \quad (82)$$

and this form of interaction is encountered in torsion gravity [20, 46]. So the origin of the torsion field seems to have a link with the spin.

Now coming to the macroscopic situation, we found Dirac particles coupling to the gravitomagnetic field of the Earth will have the interaction Hamiltonian

$$H_{max.grav.} = -\vec{S} \cdot \vec{B}_g \quad (83)$$

where \vec{B}_g is given either by (56) or (58) in the framework of Maxwellian gravity, while the interaction Hamiltonian in the framework of general relativity is considered [24] as

$$H_{GR} = -\vec{S} \cdot \vec{B}_{g,GR} \quad (84)$$

where $\vec{B}_{g,GR}$ is given by (3). Since $\vec{B}_g \neq \vec{B}_{g,GR}$ as we have seen, the two Hamiltonians (83) and (84) will be different and this difference will manifest itself in experimental tests of quantum gravity phenomena. In this connection the novel experimental proposals put forward by Camacho [15, 74] recently for quantum-mechanical detection of the gravitomagnetic field of the Earth are noteworthy.

Maxwellian Gravity is a relativistic vector model theory of gravity in flat space-time and the theory having the classical Newtonian limit is in conformity with the correspondence principle and the weak equivalence principle (WEP) at the non-relativistic level. Unlike Maxwell's equations of electromagnetic theory which represent mathematical expressions of certain experimental results, the field equations of Maxwellian Gravity represent mathematical expressions of certain theoretical deductions (without any additional postulation) from other established theories, the applicability of which to any physical situation needs verification through extensive theoretical as well as experimental work. The laws of Maxwellian gravity are valid in the world of inertial frames, while general relativity describes physics in the world of non-inertial frames. Since general relativity provides for the existence of inertial frames in the world of non-inertial frames [75], this theory is expected to find application in the area common to both general and special relativity. It is to be noted that the so called very strong Equivalence Principle [1] does not state what are the special relativistic laws of gravity although it states that for every point-like event of space-time, there exists a sufficiently small neighborhood such that in every local, freely

falling frame in that neighborhood, all laws of physics obey the laws of special relativity. Maxwellian Gravity is what represents the laws of special relativistic gravity. Although originally developed in flat-space-time, the curved space-time version of this theory is not difficult to achieve. But the present treatment of relativistic gravity in a flat space-time has two great advantages. Firstly, it provides us with a more uniform description of the gravitational field and electromagnetic fields. Secondly, it may enable us to carry out the quantization of gravitational field of this type by following the same procedure as we use for the electromagnetic field. On quantization, Maxwellian gravity with its field energy being negative and definite is expected to correspond to gravitational quanta or gravitons of vanishing rest mass and spin 1 which may produce the observed attractive type gravitational field for like charges of such fields. So from a practical point of view the theory presented here may be very useful particularly in respect of its quantization. Further study of the Generalized Dirac Equation, viz. Eq.(72), suggested in this work may lead to new insights.

As regards the “three or four crucial tests” [1, 29, 30] of general relativity we are close to get an alternative explanation for the so called Non-Newtonian excess precession of Mercury and other planets within the framework of Maxwellian Gravity. This problem will be addressed separately in our future works.

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